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Multiple fundamental strings and waves to non-linear order in the background fields

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ABSTRACT

The Chern-Simons actions of the multiple fundamental string and the multiple gravitational wave are established to full order in the background fields. Gauge invariance is checked. Special attention is drawn to the non-Abelian gauge transformations of the world-volume fields.

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1 Introduction

Multiple branes provide plenty of strange physics. They account for brane solutions with fuzzy geometry and microscopic descriptions of giant gravitons (see eg. [1, 2, 3, 4]). A multiple brane consists of N branes lying on top of each other [5]. The separation between the branes is of order of the string length. Strings stretching between them give rise to new massless modes, in addition to the modes of the strings going from a brane to itself. These extra modes fill out the $U(1)^N$ symmetry of the separate branes into a $U(N)$ symmetry group, giving the brane-stack a non-Abelian structure. At the worldvolume level the non-Abelian structure is represented by the N Born-Infeld vectors combined into one $U(N)$ vector V, and the transverse scalars enhanced to matrix coordinates X^μ transforming under the adjoint of $U(N)$. Background fields are function of the matrix coordinates via a non-Abelian Taylor expansion[6, 7]:

$$C_{\mu\nu}(x^a, X^i) = \sum_n \frac{1}{n!} \partial_{k_1} ... \partial_{k_n} C_{\mu\nu}(x^a, x^i)|_{x^i=0} \ X^{k_1} ... \ X^{k_n}. \tag{1.1}$$

Constructing a Born-Infeld action adapted to the multiple brane case is a highly non-trivial problem. The Chern-Simons action however keeps a simple structure [2, 8, 9], besides showing several intricate properties. One of these is the appearance of couplings proportional to a commutator of transverse scalars[1]. These extra, purely non-Abelian couplings allow the brane to interact dielectrically with background fields of higher rank than the brane dimension. The multiple brane then expands into a higher-dimensional, fuzzy geometry.

The multiple D-brane action and its gauge properties were studied in[10, 11, 12, 13]. An important observation was made in [12]: the matrix coordinates are affected by gauge transformations with the NS-NS parameter Σ_1. This transformation is proportional to a commutator, such that it vanishes in the Abelian limit. A consequence is that every background field will undergo this non-Abelian NS-NS transformation as well, since the fields depend on the transverse coordinates.

Less is known about multiple fundamental strings and multiple gravitational waves. The Chern-Simons actions of both objects were established to linear order in[14, 15, 4]. The multiple wave turned out to be a microscopic description of a giant graviton.

The aim of this paper is to construct the Chern-Simons action of the multiple fundamental string and the gravitational wave, both in IIA as well as in IIB, to non-linear order in the background fields. The construction is made by duality relations and checked on gauge invariance. We will follow the duality chain used in [14, 15, 4]. An overview of the dualisations is given in Figure 1.

First we uplift the Myers $D0$ brane action to eleven dimensions, where it represents a gravitational wave (W11). This action is reduced along a direction other than the Taub-NUT direction of the wave, which results in a Type IIA wave (WA). T-duality yield a Type IIB fundamental string (FB) when performed along the isometry direction. Type IIB wave (WB) with two isometry directions results from a T-duality along an other direction. The Type IIA string (FA) can be reached from two directions: either by T-dualizing the IIB string, or by T-dualizing the IIB wave. The chain closes, providing a powerful check on the duality calculations. Notice that the duality chain leads to an effective S-duality transformation. Indeed, the IIB fundamental string is S-dual to the D string, which is only a T-duality away from our starting point, the $D0$. Though the non-perturbative S-duality is not well defined in the case of multiple branes, the somewhat more rigourous chain here leads to the same result as a naive S-duality would.

Gauge transformations are well defined for the D-brane background fields. We will dualize the transformations as well as the fields to become corresponding gauge transformations for the exotic fields appearing in the multiple strings and waves. We will follow especially the transformations of the world-volume fields. The non-Abelian NS-NS transformation of the coordinate will dualise into a new transformation, following the transformations of the Born-Infeld vector and the exchange of the two-form fields B_2 and C_2.

The 11-dimensional wave is studied in section 2. The next section concentrates the reduction and the IIA wave. The IIB and IIA string actions are derived in section 4 and 5, respectively. The last action, the

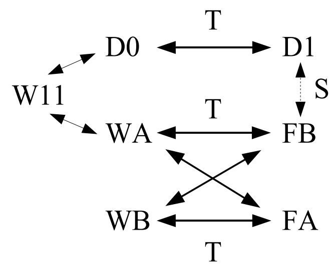

Figure 1: The various objects: the D-string (D1), the D0 brane (D0), the 11D wave (W11), the fundamental strings in IIA (FA) and IIB (FB); and the waves in IIA (WA) and IIB (WB). T-dualities are shown in solid thick lines, an uplifting/reduction relation in a thin solid line, and S-duality in dotted line.

IIB wave, appears in section 6. The appendix lists the T-duality relations and the gauge transformations of the fields appearing in the various actions.

Gauge transformations are well defined for D-brane background fields. We will here dualize the transformations as well as the fields to become corresponding gauge transformations for the exotic fields appearing in the multiple strings and waves. We will follow especially the transformations of the world-volume fields. The non-Abelian NS-NS transformation of the coordinate will dualise into new transformations, following the transformations of the Born-Infeld vector and the exchange of the two-form fields B_2 and C_2.

2 The eleven-dimensional multiple wave

Our starting point is the Myers action for $D0$-branes

$$\mathcal{L}_{D0} \quad = \quad \sum_{n=0}^{4}\sum_{k=0}^{[n]} \frac{(-i)^n}{n!} P(\mathrm{i}_X\mathrm{i}_X)^n \frac{(-)^k(2n+1)!}{2^k k!(2n-2k+1)!} C_{2n-2k+1} B^k. \tag{2.1}$$

Here $(\mathrm{i}_X\mathrm{i}_X)$ means inclusion with transverse scalars: $(\mathrm{i}_X\mathrm{i}_X)C_2 = \frac{1}{2}[X^\mu, X^\nu]C_{\nu\mu}$. The R-R gauge invariance of this action is thorougly discussed in [11, 13]. As mentioned in [12] the NS-NS variation also affects the worldvolume fields, i.e. the transverse scalars X^i and the Born-Infeld vector V_a.

$$\begin{aligned} \delta_\Sigma V_a &= -\Sigma_\mu D X^\mu \\ \delta_\Sigma X^\mu &= i\Sigma_\rho [X^\rho, X^\mu] \end{aligned} \tag{2.2}$$

Since the $D0$ brane can not carry the two-form field strength $F = 2\partial V + i[V,V]$ of the Born-Infeld vector, the latter appears only in the covariant pull-back and represents no physical degrees of freedom.

We use the multiple $D0$ action (2.1) and the uplifting rules listed in the appendix to get to the full eleven-dimensional wave action[2]. Of special concern here is the appearance of the eleventh dimension as an isometry direction. The sigma-model is then gauged with respect to this isometry, which will be expressed using the Killing vector $\hat{k}^\mu$:

$$\hat{k}^\mu = \delta^\mu{}_{11} \quad , \qquad \hat{k}_\mu = \hat{g}_{\mu 11}. \tag{2.3}$$

An inclusion of a form with the Killing vector extracts the components which have one index equal to 11. The notation is the same as for the inclusions with transverse scalars:

$$\mathrm{i}_k \hat{C}_p \quad = \quad \hat{k}^\rho \hat{C}_{\rho\mu_2\ldots\mu_p} = \hat{C}_{11,\mu_2\ldots\mu_p}. \tag{2.4}$$

[2] Eleven-dimensional fields are denoted by a hatted letter.

As a consequence, the parts of the metric corresponding to the Kaluza-Klein vector and scalar can be seen as a function of the Killing vector and its length:

$$\hat{k}^2 = \hat{k}_\mu \hat{k}^\mu = \hat{g}_{11,11} \qquad \hat{k}^{-2}\hat{k}_\mu = \frac{\hat{g}_{\mu 11}}{\hat{g}_{11,11}}. \tag{2.5}$$

The Kaluza-Klein vector $\hat{k}^{-2}\hat{k}_\mu$ reduces to the R-R vector C_1. This means that the C_1 variation with parameter Λ_0 should correspond to a coordinate transformation of the eleventh dimension.

The action of the eleven-dimensional wave then becomes:

$$\begin{aligned}\mathcal{L}_{W11} \;=\;& STr\Big\{P\Big(\hat{k}^{-2}\hat{k}_1\Big) - iP(\mathrm{i}_X\mathrm{i}_X)\Big(\hat{C}_3 - 3\mathrm{i}_k\hat{C}_3\hat{k}^{-2}\hat{k}_1\Big) \\ & -\frac{1}{2}P(\mathrm{i}_X\mathrm{i}_X)^2\Big(\mathrm{i}_k\hat{C}_6 - 5\hat{C}_3\mathrm{i}_k\hat{C}_3 + 15\mathrm{i}_k\hat{C}_3\mathrm{i}_k\hat{C}_3\hat{k}^{-2}\hat{k}_1\Big) \\ & +\frac{i}{6}P(\mathrm{i}_X\mathrm{i}_X)^3\Big(\mathrm{i}_k\hat{C}_8 - 21\mathrm{i}_k\hat{C}_6\mathrm{i}_k\hat{C}_3 + 35\hat{C}_3\mathrm{i}_k\hat{C}_3\mathrm{i}_k\hat{C}_3 - 105\hat{C}_3\mathrm{i}_k\hat{C}_3 i_k\hat{C}_3\hat{k}^{-2}\hat{k}_1\Big) \\ & +\frac{1}{24}P(\mathrm{i}_X\mathrm{i}_X)^4\Big(\mathrm{i}_k\hat{C}_{10} - 36\mathrm{i}_k\hat{C}_8\mathrm{i}_k\hat{C}_3 + 378\mathrm{i}_k\hat{C}_6\mathrm{i}_k\hat{C}_3\mathrm{i}_k\hat{C}_3 - 315\hat{C}_3\mathrm{i}_k\hat{C}_3\mathrm{i}_k\hat{C}_3\mathrm{i}_k\hat{C}_3 \\ & \qquad +945\mathrm{i}_k\hat{C}_3\mathrm{i}_k\hat{C}_3\mathrm{i}_k\hat{C}_3\hat{k}^{-2}\hat{k}_1\Big)\Big\}.\end{aligned} \tag{2.6}$$

The appearance of the Killing vector makes the action invariant under coordinate transformations of the eleventh coordinate. In [4, 14, 15], covariant pull-backs were used to get a more Lorentz-invariant look. To non-linear order, however, the Killing vector appears not only pull-backed but also contracted with a commutator. Defining modified commutators is possible, but it is easier and more transparant to see the Killing vector as a background field.

A more compact way of writing (2.6) uses the property $\mathrm{i}_k(\hat{k}^{-2}\hat{k}_1) = \frac{g_{11,11}}{g_{11,11}} = 1$, such that for every p-form T_p:

$$(p+1)\mathrm{i}_k(\hat{T}_p\hat{k}^{-2}\hat{k}_1) \;=\; p\;\mathrm{i}_k\hat{T}_p\;\hat{k}^{-2}\hat{k}_1 + (-)^p\hat{T}_p. \tag{2.7}$$

This proves to be useful, especially when dealing with multiple isometry directions, such as in the IIB multiple wave. Written in this way, the action (2.6) becomes:

$$\begin{aligned}\mathcal{L}_{W11} \;=\;& STr\Big\{P\Big(\hat{k}^{-2}\hat{k}_1\Big) + iP(\mathrm{i}_X\mathrm{i}_X)\mathrm{i}_k\Big(4\hat{C}_3\hat{k}^{-2}\hat{k}_1\Big) \\ & +\frac{1}{2}P(\mathrm{i}_X\mathrm{i}_X)^2\mathrm{i}_k\Big[6\Big(\mathrm{i}_k\hat{C}_6 - 5\hat{C}_3\mathrm{i}_k\hat{C}_3\Big)\hat{k}^{-2}\hat{k}_1\Big] \\ & -\frac{i}{6}P(\mathrm{i}_X\mathrm{i}_X)^3\mathrm{i}_k\Big[8\Big(\mathrm{i}_k\hat{C}_8 - 21\mathrm{i}_k\hat{C}_6\mathrm{i}_k\hat{C}_3 + 35\hat{C}_3\mathrm{i}_k\hat{C}_3\mathrm{i}_k\hat{C}_3\Big)\hat{k}^{-2}\hat{k}_1\Big] \\ & -\frac{1}{24}P(\mathrm{i}_X\mathrm{i}_X)^4\mathrm{i}_k\Big[10\Big(\mathrm{i}_k\hat{C}_{10} - 36\mathrm{i}_k\hat{C}_8\mathrm{i}_k\hat{C}_3 + 378\mathrm{i}_k\hat{C}_6\mathrm{i}_k\hat{C}_3\mathrm{i}_k\hat{C}_3 - 315\hat{C}_3\mathrm{i}_k\hat{C}_3\mathrm{i}_k\hat{C}_3\mathrm{i}_k\hat{C}_3\Big)\hat{k}^{-2}\hat{k}_1\Big]\Big\}.\end{aligned} \tag{2.8}$$

The invariance of the action under coordinate transformations in the isometry direction is now manifest, since everything is written as an inclusion with $\hat{k}_1$.

We mention here the variation of the worldvolume fields, the Born-Infeld vector V_a and the transverse scalar X^μ, which are the uplifting of the (2.2) rules:

$$\begin{aligned}\delta\hat{V}_a &= (\mathrm{i}_k\hat{\Lambda}_2)_\rho D_a X^\rho \\ \delta\hat{X}^\mu &= -i(\mathrm{i}_k\hat{\Lambda}_2)_\rho[X^\rho, X^\mu].\end{aligned} \tag{2.9}$$

Just as in the $D0$ brane action, the vector V appears only in the covariant pull-back and represents no physical degrees of freedom. Indeed, no Born-Infeld vector can exist in the 11D theory because there are no fundamental strings. So the V is pure gauge and indicates how the pull-back should transform under the NS-NS transformations. Notice that the worldvolume fields now vary with $\mathrm{i}_k\hat{\Lambda}_2$. We can already see that the NS-NS variation of the worldvolume fields are dualized into R-R transformations. The parameter Σ_1 from the $D0$ brane is uplifted to $\mathrm{i}_k\hat{\Lambda}_2$. When we reduce along a non-isometry direction to the IIA multiple wave, the parameter reduces to $\mathrm{i}_k\Lambda_2$. The parameter will remain of the R-R kind throughout the subsequent T-dualities.

3 The IIA multiple wave

When reducing the action (2.8) along a direction other than the Taub-NUT direction, one finds the Chern-Simons action for the IIA multiple wave. This was carried out to linear order in [15]. Using the full reduction rules we get the following result.

We assume that the reduced dimension is orthogonal to the Taub-NUT direction of the wave. This restriction is necessary to close the duality chain. Closure requires the T-dualities to commute, which is the case for orthogonal directions. The orthogonality is assured by setting $\mathrm{i}_z(\hat{k}^{-2}\hat{k}_1) = \frac{g_{z11}}{g_{11,11}}$ to zero. $\hat{k}^{-2}\hat{k}_1$ reduces then to $k^{-2}k_1 = \frac{g_{\mu 11}}{g_{11,11}}$.

The reduced coordinate X^z becomes a scalar which we call S. So

$$X^z \rightarrow S \qquad\qquad DX^z \rightarrow DS. \tag{3.1}$$

With these assumptions the Type IIA wave action becomes:

$$\begin{aligned}
\mathcal{L}_{WA} \;=\; & \Big\{P\Big(k^{-2}k_1\Big) - i\left(DS(\mathrm{i}_X\mathrm{i}_X) - P\mathrm{i}_{[S,X]}\right)\mathrm{i}_k\Big(3B_2k^{-2}k_1\Big) \qquad (3.2)\\
& +iP(\mathrm{i}_X\mathrm{i}_X)\mathrm{i}_k\Big(4C_3k^{-2}k_1\Big) + \frac{1}{2}\left(DS(\mathrm{i}_X\mathrm{i}_X)^2 - 2P(\mathrm{i}_X\mathrm{i}_X)\mathrm{i}_{[S,X]}\right)\mathrm{i}_k\Big[5(i_kC_5 + 4C_3i_kB_2)k^{-2}k_1\Big]\\
& +\frac{1}{2}P(\mathrm{i}_X\mathrm{i}_X)^2\mathrm{i}_k\Big[6\Big(\mathrm{i}_kB_6 - 5C_3\mathrm{i}_kC_3\Big)k^{-2}k_1\Big]\\
& -\frac{i}{6}\left(DS(\mathrm{i}_X\mathrm{i}_X)^3 - 3P(\mathrm{i}_X\mathrm{i}_X)^2\mathrm{i}_{[S,X]}\right)\mathrm{i}_k\Big[7\Big(\mathrm{i}_kN_7 + 6\mathrm{i}_kB_6\mathrm{i}_kB_2 - 15\mathrm{i}_kC_5\mathrm{i}_kC_3\\
& \qquad -30C_3\mathrm{i}_kC_3\mathrm{i}_kB_2\Big)k^{-2}k_1\Big]\\
& -\frac{i}{6}P(\mathrm{i}_X\mathrm{i}_X)^3\mathrm{i}_k\Big[8\Big(\mathrm{i}_kN_8 - 21\mathrm{i}_kB_6\mathrm{i}_kC_3 + 35C_3(\mathrm{i}_kC_3)^2\Big)k^{-2}k_1\Big]\\
& -\frac{1}{24}\left(DS(\mathrm{i}_X\mathrm{i}_X)^4 - 4P(\mathrm{i}_X\mathrm{i}_X)^3i_{[S,X]}\right)i_k\Big[9\Big(\mathrm{i}_kN_9 + 8\mathrm{i}_kN_8\mathrm{i}_kB_2 - 28\mathrm{i}_kN_7\mathrm{i}_kC_3\\
& \qquad -168\mathrm{i}_kB_6\mathrm{i}_kC_3\mathrm{i}_kB_2 + 210\mathrm{i}_kC_5(\mathrm{i}_kC_3)^2 + 280C_3(\mathrm{i}_kC_3)^2\mathrm{i}_kB_2\Big)k^{-2}k_1\Big]\\
& -\frac{1}{24}P(\mathrm{i}_X\mathrm{i}_X)^4\mathrm{i}_k\Big[10\Big(\mathrm{i}_kQ_{10} - 36\mathrm{i}_kN_8\mathrm{i}_kC_3 + 378\mathrm{i}_kB_6(\mathrm{i}_kC_3)^2 - 315C_3(\mathrm{i}_kC_3)^3\Big)k^{-2}k_1\Big]\Big\}
\end{aligned}$$

Again the action is invariant under the gauge transformations. Since $\mathrm{i}_k\hat{\Lambda}_2$ reduces to both $\mathrm{i}_k\Lambda_2$ and $\mathrm{i}_k\Sigma_1$, we will encounter these two parameters in the variation of the Born-Infeld vector, the transverse scalars and the scalar S. Unlike in the case of the 11D theory, a Born-Infeld vector is possible here. However, due to lack of worldvolume dimensions no Born-Infeld field strength appears. So for the IIA wave too the Born-Infeld field is pure gauge. This will be the same for the IIB gravitational wave, but not for the IIA and IIB fundamental strings. Indeed, the latter have two worldvolume dimensions, making a nonvanishing Born-Infeld field strength possible.

The worldvolume field variations are:

$$\delta V_a \;=\; D_aX^\rho(\mathrm{i}_k\Lambda_2)_\rho + D_aS\mathrm{i}_k\Sigma_1 \tag{3.3}$$

$$
\begin{aligned}
\delta X^\mu &= -i(\mathrm{i}_k\Lambda_2)_\rho[X^\rho, X^\mu] - i\mathrm{i}_k\Sigma_1[S, X^\mu] \\
\delta S &= -\Lambda_0 - i(\mathrm{i}_k\Lambda_2)_\rho[X^\rho, S].
\end{aligned}
$$

4 The multiple $F1$ in type IIB

When T-dualized along its Taub-NUT direction, the IIA gravitational wave becomes a fundamental string in Type IIB. Hereby the worldvolume fields are mapped as usual. The scalar S becomes a component of the Born-Infeld vector:

$$
S \quad \longleftrightarrow \quad -V_x. \tag{4.1}
$$

The duality was carried out to linear order in [14]. Again, using the reduction and uplifting rules on the action (3.2) we recover a fully gauge invariant action of the multiple fundamental string in Type IIB.

$$
\begin{aligned}
\mathcal{L}_{FB} = \; & STr\Big\{P(B_2) + i(\mathrm{i}_X\mathrm{i}_X)(B_2)\wedge F + iP(\mathrm{i}_X\mathrm{i}_X)(C_4) - \frac{1}{2}(\mathrm{i}_X\mathrm{i}_X)^2(C_4)\wedge F \\
& + \frac{1}{2}P(\mathrm{i}_X\mathrm{i}_X)^2(B_6) + \frac{i}{6}(\mathrm{i}_X\mathrm{i}_X)^3(B_6)\wedge F - \frac{i}{6}P(\mathrm{i}_X\mathrm{i}_X)^3(Q_8) \\
& + \frac{1}{24}(\mathrm{i}_X\mathrm{i}_X)^4(Q_8)\wedge F - \frac{1}{24}P(\mathrm{i}_X\mathrm{i}_X)^4(Q_{10})\Big\}
\end{aligned} \tag{4.2}
$$

Notice that there appear no isometry directions any more. The multiple fundamental string in IIB has the full ten-dimensional Lorentz-invariance, just like a single fundamental string.

As for the variation parameters, T-duality maps both Λ_0 and $\mathrm{i}_k\Lambda_2$ onto $\mathrm{i}_k\Lambda_1$, following the recombination of the IIA fields V_a and S into the IIB Born-Infeld vector V_a. The $\mathrm{i}_k\Sigma_1$ variation becomes a coordinate transformation. So the gauge variation of the worldvolume fields goes only with Λ_1.

$$
\begin{aligned}
\delta V_a &= -\Lambda_\rho D_a X^\rho \\
\delta X^\mu &= i\Lambda_\rho[X^\rho, X^\mu]
\end{aligned} \tag{4.3}
$$

Now it is very clear how the twoforms are interchanged with respect to the D-string. The worldvolume fields vary with the R-R parameter. As was already mentioned in [14], the Born-Infeld vector of the D-branes is changed by S-duality into another worldvolume vector. In the Abelian limit the field strength of this vector combines with the pull-backed R-R twoform to an invariant field strength $\mathcal{F}' = 2\partial V' + P[C_2]$.

5 Transverse T-duality: a type IIA fundamental string

As mentioned in [14], a T-dualization of the type IIB fundamental string gives a fundamental string with winding number in type IIA.

$$
\begin{aligned}
\mathcal{L}_{FA} = \; & STr\{\left(-2D\omega P + iF\mathrm{i}_{[\omega,X]}\right)\Big(l^{-2}l_1\Big) + \left(P + iF(\mathrm{i}_X\mathrm{i}_X)\right)\mathrm{i}_l\Big(3B_2 l^{-2}l_1\Big) \\
& + i\left(-2D\omega P(\mathrm{i}_X\mathrm{i}_X) + P\mathrm{i}_{[\omega,X]} + iF(\mathrm{i}_X\mathrm{i}_X)\mathrm{i}_{[\omega,X]}\right)\mathrm{i}_l\Big(4C_3 l^{-2}l_1\Big) \\
& + i\left(P(\mathrm{i}_X\mathrm{i}_X) + \frac{i}{2}F(\mathrm{i}_X\mathrm{i}_X)^2\right)\mathrm{i}_l\Big[5\Big(\mathrm{i}_l C_5 + 4C_3\mathrm{i}_l B_2\Big)l^{-2}l_1\Big] \\
& + \frac{1}{2}\left(-2D\omega P(\mathrm{i}_X\mathrm{i}_X)^2 + 2P(\mathrm{i}_X\mathrm{i}_X)i_{[\omega,X]} + iF(\mathrm{i}_X\mathrm{i}_X)^2\mathrm{i}_{[\omega,X]}\right)\mathrm{i}_l\Big[6\Big(\mathrm{i}_l B_6 - 5C_3\mathrm{i}_l C_3\Big)l^{-2}l_1\Big]
\end{aligned} \tag{5.1}
$$

$$+\frac{1}{2}\left(P(\mathrm{i}_X\mathrm{i}_X)^2+\frac{i}{3}F(\mathrm{i}_X\mathrm{i}_X)^3\right)\mathrm{i}_l\Big[7\Big(\mathrm{i}_lN_7+6\mathrm{i}_lB_6\mathrm{i}_lB_2-15\mathrm{i}_lC_5\mathrm{i}_lC_3-30C_3\mathrm{i}_lC_3\mathrm{i}_lB_2\Big)l^{-2}l_1\Big]$$
$$-\frac{i}{6}\left(-2D\omega P(\mathrm{i}_X\mathrm{i}_X)^3+3P(\mathrm{i}_X\mathrm{i}_X)^2\mathrm{i}_{[\omega,X]}+iF(\mathrm{i}_X\mathrm{i}_X)^3\mathrm{i}_{[\omega,X]}\right)$$
$$\mathrm{i}_l\Big[8\Big(\mathrm{i}_lN_8-21\mathrm{i}_lB_6\mathrm{i}_lC_3+35C_3(\mathrm{i}_lC_3)^2\Big)l^{-2}l_1\Big]$$
$$-\frac{i}{6}P(\mathrm{i}_X\mathrm{i}_X)^3\mathrm{i}_l\Big[9\Big(\mathrm{i}_lN_9+8\mathrm{i}_lN_8\mathrm{i}_lB_2-28\mathrm{i}_lN_7\mathrm{i}_lC_3-168\mathrm{i}_lB_6i_lC_3\mathrm{i}_lB_2+210\mathrm{i}_lC_5(\mathrm{i}_lC_3)^2$$
$$+280C_3(\mathrm{i}_lC_3)^2\mathrm{i}_lB_2\Big)l^{-2}l_1\Big]$$
$$-\frac{1}{6}P(\mathrm{i}_X\mathrm{i}_X)^3i_{[\omega,X]}\mathrm{i}_l\Big[10\Big(\mathrm{i}_lQ_{10}-36\mathrm{i}_lN_8\mathrm{i}_lC_3+378\mathrm{i}_lB_6(\mathrm{i}_lC_3)^2-315C_3(\mathrm{i}_lC_3)^3\Big)l^{-2}l_1\Big]\Big\}.$$

The T-duality rules are the same as the ones used in Section 4. Indeed, both dualities connect a IIB theory to the IIA with one isometry direction. To make clear that the isometry directions fulfill a different role in the IIA string and the IIA wave, we denote their Killing vectors with different symbols. So k is the IIA wave's Taub-NUT direction, while l denotes the string's wrapping direction. In both cases the isometry directions allow for exotic form fields.

The worldvolume gauge variations are now

$$\begin{aligned}\delta V_a &= (i_l\Lambda_2)_\rho D_aX^\rho+\Lambda_0D_a\omega \\ \delta X^\mu &= -i(i_l\Lambda_2)_\rho[X^\rho,X^\mu]-i\Lambda_0[\omega,X^\mu] \\ \delta\omega &= -\mathrm{i}_l\Sigma_1-i(\mathrm{i}_l\Lambda_2)_\rho[X^\rho,\omega].\end{aligned} \tag{5.2}$$

6 The Type IIB gravitational wave

The IIB gravitational wave can be reached by a transverse T-duality on the IIA wave, but also via a T-duality along a worldvolume direction of the IIA fundamental string. This two ways should give the same result, closing the chain of dualities. This is indeed the case, up to a sign difference of the R-R fields. This sign difference comes from interchanging the two T-duality directions, as mentioned in [15]. The action mentioned here is the one coming from the transverse T-duality on the IIA wave, in order to compare with [15]. In addition to the duality rules we used for the IIB string, we need now more dualities connecting to the exotic IIB fields with two isometries. Due to our assumption of orthogonal isometry directions, we have that $\mathrm{i}_l(k^{-2}k_1)=\mathrm{i}_k(l^{-2}l_1)=\mathrm{i}_k\mathrm{i}_lB_2=0$. This taken into account, the IIB wave action is:

$$\mathcal{L}_{WB} = STr\Big\{i\left(D\omega\mathrm{i}_{[S,X]}-DS\mathrm{i}_{[\omega,X]}-[S,\omega]P\right)(l^{-2}l_1)+P(k^{-2}k_1) \tag{6.1}$$
$$+i\left(DS(\mathrm{i}_X\mathrm{i}_X)-P\mathrm{i}_{[S,X]}\right)\mathrm{i}_k\mathrm{i}_l\Big(12B_2k^{-2}k_1l^{-2}l_1\Big)$$
$$+i\left(D\omega(\mathrm{i}_X\mathrm{i}_X)-P\mathrm{i}_{[\omega,X]}\right)\mathrm{i}_k\mathrm{i}_l\Big(12C_2k^{-2}k_1l^{-2}l_1\Big)$$
$$+\left(D\omega(\mathrm{i}_X\mathrm{i}_X)\mathrm{i}_{[S,X]}-DS(\mathrm{i}_X\mathrm{i}_X)\mathrm{i}_{[\omega,X]}+P\mathrm{i}_{[S,X]}\mathrm{i}_{[\omega,X]}-[S,\omega]P(\mathrm{i}_X\mathrm{i}_X)\right)$$
$$\mathrm{i}_k\mathrm{i}_l\Big[20\Big(\mathrm{i}_kC_4-3C_2\mathrm{i}_kB_2\Big)k^{-2}k_1l^{-2}l_1\Big]$$
$$+iP(\mathrm{i}_X\mathrm{i}_X)\mathrm{i}_k\mathrm{i}_l\Big[20\Big(\mathrm{i}_lC_4-3C_2\mathrm{i}_lB_2\Big)k^{-2}k_1l^{-2}l_1\Big]$$
$$-\frac{1}{2}\left(DS(\mathrm{i}_X\mathrm{i}_X)^2-2P(\mathrm{i}_X\mathrm{i}_X)\mathrm{i}_{[S,X]}\right)\mathrm{i}_k\mathrm{i}_l\Big[30\Big(\mathrm{i}_k\mathrm{i}_lC_6+4\mathrm{i}_lC_4\mathrm{i}_kB_2-4\mathrm{i}_kC_4\mathrm{i}_lB_2$$
$$-12C_2\mathrm{i}_lB_2\mathrm{i}_kB_2\Big)k^{-2}k_1l^{-2}l_1\Big]$$
$$-\frac{1}{2}\left(D\omega(\mathrm{i}_X\mathrm{i}_X)^2-2P(\mathrm{i}_X\mathrm{i}_X)\mathrm{i}_{[\omega,X]}\right)\mathrm{i}_k\mathrm{i}_l\Big[30\Big(\mathrm{i}_k\mathrm{i}_lB_6+6\mathrm{i}_k\mathrm{i}_lC_4C_2\Big)k^{-2}k_1l^{-2}l_1\Big]$$

$$
\begin{aligned}
&-\frac{i}{2}\left(D\omega(\mathrm{i}_X\mathrm{i}_X)^2\mathrm{i}_{[S,X]} - DS(\mathrm{i}_X\mathrm{i}_X)^2\mathrm{i}_{[\omega,X]} + 2P(\mathrm{i}_X\mathrm{i}_X)\mathrm{i}_{[S,X]}\mathrm{i}_{[\omega,X]} - [S,\omega]P(\mathrm{i}_X\mathrm{i}_X)^2\right)\\
&\quad \mathrm{i}_k\mathrm{i}_l\Big[42\Big(\mathrm{i}_k\mathrm{i}_l N_7 + 5\mathrm{i}_k\mathrm{i}_l B_6\mathrm{i}_k B_2 - 5\mathrm{i}_k C_4\mathrm{i}_k\mathrm{i}_l C_4 + 30\mathrm{i}_k\mathrm{i}_l C_4 C_2\mathrm{i}_k B_2\Big)k^{-2}k_1 l^{-2}l_1\Big]\\
&+\frac{1}{2}P(\mathrm{i}_X\mathrm{i}_X)^2\mathrm{i}_k\mathrm{i}_l\Big[42\Big(\mathrm{i}_k\mathrm{i}_l\mathcal{N}_7 - 5\mathrm{i}_l C_4\mathrm{i}_k\mathrm{i}_l C_4 + 5\mathrm{i}_k\mathrm{i}_l B_6\mathrm{i}_l B_2 + 30\mathrm{i}_k\mathrm{i}_l C_4 C_2\mathrm{i}_l B_2\Big)k^{-2}k_1 l^{-2}l_1\Big]\\
&+\frac{i}{6}\left(DS(\mathrm{i}_X\mathrm{i}_X)^3 - 3P(\mathrm{i}_X\mathrm{i}_X)^2\mathrm{i}_{[S,X]}\right)\mathrm{i}_k\mathrm{i}_l\Big[56\Big(\mathrm{i}_k\mathrm{i}_l\mathcal{N}_8 + 6\mathrm{i}_k\mathrm{i}_l\mathcal{N}_7\mathrm{i}_k B_2 - 6\mathrm{i}_k\mathrm{i}_l N_7\mathrm{i}_l B_2\\
&\qquad -30\mathrm{i}_k\mathrm{i}_l B_6\mathrm{i}_k B_2\mathrm{i}_l B_2 - 180\mathrm{i}_k\mathrm{i}_l C_4 C_2\mathrm{i}_k B_2\mathrm{i}_l B_2 - 15\mathrm{i}_k\mathrm{i}_l C_4\mathrm{i}_k\mathrm{i}_l C_6\\
&\qquad -30\mathrm{i}_l C_4\mathrm{i}_k\mathrm{i}_l C_4\mathrm{i}_k B_2 + 30\mathrm{i}_k C_4\mathrm{i}_k\mathrm{i}_l C_4\mathrm{i}_l B_2\Big)k^{-2}k_1 l^{-2}l_1\Big]\\
&+\frac{i}{6}\left(D\omega(\mathrm{i}_X\mathrm{i}_X)^3 - 3P(\mathrm{i}_X\mathrm{i}_X)^2\mathrm{i}_{[\omega,X]}\right)\mathrm{i}_k\mathrm{i}_l\Big[56\Big(\mathrm{i}_k\mathrm{i}_l N_8 - 45C_2\mathrm{i}_k\mathrm{i}_l C_4\mathrm{i}_k\mathrm{i}_l C_4\Big)k^{-2}k_1 l^{-2}l_1\Big]\\
&-\frac{1}{6}\left(D\omega(\mathrm{i}_X\mathrm{i}_X)^3\mathrm{i}_{[S,X]} - DS(\mathrm{i}_X\mathrm{i}_X)^3\mathrm{i}_{[\omega,X]} + 3P(\mathrm{i}_X\mathrm{i}_X)^2\mathrm{i}_{[S,X]}\mathrm{i}_{[\omega,X]} - [S,\omega]P(\mathrm{i}_X\mathrm{i}_X)^3\right)\\
&\quad \mathrm{i}_k\mathrm{i}_l\Big[72\Big(\mathrm{i}_k\mathrm{i}_l N_9 + 7\mathrm{i}_k\mathrm{i}_l N_8\mathrm{i}_k B_2 - 21\mathrm{i}_k\mathrm{i}_l N_7\mathrm{i}_k\mathrm{i}_l C_4\\
&\qquad +35\mathrm{i}_k C_4(\mathrm{i}_k\mathrm{i}_l C_4)^2 - 315C_2(\mathrm{i}_k\mathrm{i}_l C_4)^2\mathrm{i}_k B_2\Big)k^{-2}k_1 l^{-2}l_1\Big]\\
&-\frac{i}{6}P(\mathrm{i}_X\mathrm{i}_X)^3\mathrm{i}_k\mathrm{i}_l\Big[72\Big(\mathrm{i}_k\mathrm{i}_l\mathcal{N}_9 + 7\mathrm{i}_k\mathrm{i}_l N_8\mathrm{i}_l B_2 - 21\mathrm{i}_k\mathrm{i}_l\mathcal{N}_7\mathrm{i}_k\mathrm{i}_l C_4 + 35\mathrm{i}_l C_4(\mathrm{i}_k\mathrm{i}_l C_4)^2\\
&\qquad -315C_2(\mathrm{i}_k\mathrm{i}_l C_4)^2\mathrm{i}_l B_2\Big)k^{-2}k_1 l^{-2}l_1\Big]\\
&-\frac{1}{6}P(\mathrm{i}_X\mathrm{i}_X)^3\mathrm{i}_{[S,X]}\mathrm{i}_k\mathrm{i}_l\Big[90\Big(\mathrm{i}_k\mathrm{i}_l N_{10} - 8\mathrm{i}_k\mathrm{i}_l N_9\mathrm{i}_l B_2 + 8\mathrm{i}_k\mathrm{i}_l\mathcal{N}_9\mathrm{i}_k B_2 - 56\mathrm{i}_k\mathrm{i}_l N_8\mathrm{i}_k B_2\mathrm{i}_l B_2\\
&\qquad -28\mathrm{i}_k\mathrm{i}_l\mathcal{N}_8\mathrm{i}_k\mathrm{i}_l C_4 - 168\mathrm{i}_k\mathrm{i}_l\mathcal{N}_7\mathrm{i}_k\mathrm{i}_l C_4\mathrm{i}_k B_2 + 168\mathrm{i}_k\mathrm{i}_l N_7\mathrm{i}_k\mathrm{i}_l C_4\mathrm{i}_l B_2\\
&\qquad +210\mathrm{i}_k\mathrm{i}_l C_6(\mathrm{i}_k\mathrm{i}_l C_4)^2 - 280\mathrm{i}_k C_4(\mathrm{i}_k\mathrm{i}_l C_4)^2\mathrm{i}_l B_2 + 280\mathrm{i}_l C_4(\mathrm{i}_k\mathrm{i}_l C_4)^2\mathrm{i}_k B_2\\
&\qquad +2520C_2(\mathrm{i}_k\mathrm{i}_l C_4)^2\mathrm{i}_k B_2\mathrm{i}_l B_2\Big)k^{-2}k_1 l^{-2}l_1\Big]\\
&-\frac{1}{6}P(\mathrm{i}_X\mathrm{i}_X)^3\mathrm{i}_{[\omega,X]}\mathrm{i}_k\mathrm{i}_l\Big[90\Big(\mathrm{i}_k\mathrm{i}_l\mathcal{N}_{10} + 420C_2(\mathrm{i}_k\mathrm{i}_l C_4)^3\Big)k^{-2}k_1 l^{-2}l_1\Big]\Big\}.
\end{aligned}
$$

Not only the action but also the transformations get a more complicated form due to the two isometries. T-duality of the IIA transformations learns us that three parameters affect the Born-Infeld field and the transverse scalars, while the scalars ω and S shift with two more parameters.

$$
\begin{aligned}
\delta V_a &= -(\mathrm{i}_k\mathrm{i}_l\Lambda_3)_\rho D_a X^\rho + (\mathrm{i}_k\Lambda_1)D_a\omega + (\mathrm{i}_k\Sigma_1)D_a S\\
\delta X^\mu &= i(\mathrm{i}_k\mathrm{i}_l\Lambda_3)_\rho[X^\rho, X^\mu] - i(\mathrm{i}_k\Lambda_1)[\omega, X^\mu] - i(\mathrm{i}_k\Sigma_1)[S, X^\mu]\\
\delta\omega &= -\mathrm{i}_l\Sigma_1 - i(\mathrm{i}_k\mathrm{i}_l\Lambda_3)_\rho[X^\rho,\omega] - (\mathrm{i}_k\Sigma_1)[S,\omega]\\
\delta S &= \mathrm{i}_l\Lambda_1 - i(\mathrm{i}_k\mathrm{i}_l\Lambda_3)_\rho[X^\rho, S] - (\mathrm{i}_k\Lambda_1)[\omega, S]
\end{aligned}
\qquad (6.2)
$$

The most remarkable feature of this action and its variation is their complexity, especially when compared to the IIB fundamental string. The reason of this lies indeed in the isometries. In fact, the IIB wave action is eight-dimensional instead of ten-dimensional. The different role played by the parameters with an i_k inclusion and an i_l inclusion points at the different nature of the isometries. k is the Taub-NUT direction while l is the wrapping direction. $\mathrm{i}_k\mathrm{i}_l\Lambda_3$ is the wave's counterpart of Λ_1. All other variations appear only due to the isometries. Indeed, they are always proportional to the scalars S and ω.

7 Discussion

In this paper we have discussed fully gauge invariant actions for the multiple 11D gravitational wave, the Type IIA and IIB wave and the IIA and IIB string. Special attention is drawn to the world volume fields and their non-Abelian gauge variations.

Another issue, not yet mentioned, is the role a mass parameter would take. Dielectric D-branes in massive Type IIA have been studied in [11]. Two ways can be followed to extend the Type IIA actions here with a mass: either perform a massive T-duality on the Type IIB string, or start with the $D0$ brane in massive Type IIA and following the chain used here again. These two methods will not yield the same result. The duality chain $D0 \leftarrow W11 \rightarrow WA$ interchanges the fields B_2 and $\mathrm{i}_k C_3$. If we uplift a multiple $D0$-brane in Romans theory to 11 dimensions, we will end up with a multiple gravitational wave in BLO theory [16]. Reduction of this wave will result in a wave in some theory where $\mathrm{i}_k C_3$ transforms under massive gauge transformation instead of B_2. If we want to describe a multiple wave in Romans theory, massive T-duality from the FB should give the right action.

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A Reduction and T-duality rules

Here we list the reduction and T-duality rules we used. They can also be found, up to linear order, in [15, 4]. Most of them to are mentioned to higher order in [17]. Some sign conventions may differ.

Reduction rules for the eleven-dimensional fields along the isometry direction:

$$
\begin{aligned}
\hat{k}^{-2}\hat{k}_1 &= C_1 \\
\mathrm{i}_k\hat{C}_3 &= B_2 & \qquad \hat{C}_3 &= C_3 \\
\mathrm{i}_k\hat{C}_6 &= C_5 - 5C_3B_2 \\
\mathrm{i}_k\hat{C}_8 &= C_7 - 35C_3B_2B_2 \\
\mathrm{i}_k\hat{C}_{10} &= C_9 - 315C_3B_2B_2B_2 .
\end{aligned} \tag{A.1}
$$

Reduction of the eleven-dimensional fields along a direction other than the isometry direction:

$$
\begin{aligned}
\mathrm{i}_z(\hat{k}^{-2})\hat{k}_1 &= 0 & \hat{k}^{-2}\hat{k}_1 &= k^{-2}k_1 \\
\mathrm{i}_k\mathrm{i}_z\hat{C}_3 &= \mathrm{i}_kB_2 & \mathrm{i}_k\hat{C}_3 &= \mathrm{i}_kC_3 \\
\mathrm{i}_z\hat{C}_3 &= B_2 & \hat{C}_3 &= C_3 \\
\mathrm{i}_z\hat{C}_6 &= C_5 - 5C_3B_2 & \hat{C}_6 &= B_6 \\
\mathrm{i}_k\mathrm{i}_z\hat{C}_8 &= \mathrm{i}_kN_7 - 30(\mathrm{i}_kC_3)^2B_2 + 20\mathrm{i}_kC_3C_3\mathrm{i}_kB_2 & \mathrm{i}_k\hat{C}_8 &= \mathrm{i}_kN_8 \\
\mathrm{i}_k\mathrm{i}_z\hat{C}_{10} &= \mathrm{i}_kN_9 - 315(\mathrm{i}_kC_3)^3B_2 + 210C_3(\mathrm{i}_kC_3)^2\mathrm{i}_kB_2 & \mathrm{i}_k\hat{C}_{10} &= \mathrm{i}_kN_{10} .
\end{aligned} \tag{A.2}
$$

T-duality along the IIA isometry direction. The right-hand side are the IIA fields with k as isometry direction while the IIB potentials on the left-hand side have no isometry.

$$
\begin{aligned}
\frac{g_z}{g_{zz}} &= -\mathrm{i}_kB_2 \\
-\mathrm{i}_zB_2 &= k^{-2}k_1 \\
B_2 &= B_2 + 2\mathrm{i}_kB_2k^{-2}k_1 \\
\mathrm{i}_zC_{2n} &= C_{2n-1} - (2n-1)\mathrm{i}_kC_{2n-1}k^{-2}k_1 \\
C_{2n} &= \mathrm{i}_kC_{2n+1} + 2n(C_{2n-1} - (2n-1)\mathrm{i}_kC_{2n}k^{-2}k_1)\mathrm{i}_kB_2 \\
\mathrm{i}_zB_6 &= \mathrm{i}_kB_6 - 5(C_3 - 3\mathrm{i}_kC_3k^{-2}k_1)\mathrm{i}_kC_3 \\
B_6 &= \mathrm{i}_kN_7 + 6(\mathrm{i}_kB_6 - 5(C_3 - 3\mathrm{i}_kC_3k^{-2}k_1)\mathrm{i}_kC_3)\mathrm{i}_kB_2 - 15\mathrm{i}_kC_5\mathrm{i}_kC_3 \\
\mathrm{i}_zQ_8 &= \mathrm{i}_kN_8 - 21\mathrm{i}_kB_6\mathrm{i}_kC_3 + 35(C_3 - 3\mathrm{i}_kC_3k^{-2}k_1)(\mathrm{i}_kC_3)^2 \\
Q_8 &= \mathrm{i}_kN_9 + 8(\mathrm{i}_kN_8 - 21\mathrm{i}_kB_6\mathrm{i}_kC_3 + 35(C_3 - 3\mathrm{i}_kC_3k^{-2}k_1)(\mathrm{i}_kC_3)^2)\mathrm{i}_kB_2 \\
&\quad -27\mathrm{i}_kN_7\mathrm{i}_kC_3 + 210\mathrm{i}_kC_5(\mathrm{i}_kC_3)^2 \\
i_zQ_{10} &= i_kN_{10} - 36\mathrm{i}_kN_8\mathrm{i}_kC_3 + 378\mathrm{i}_kB_6(\mathrm{i}_kC_3)^2 - 315(C_3 - \mathrm{i}_kC_3k^{-2}k_1)(\mathrm{i}_kC_3)^3
\end{aligned} \tag{A.3}
$$

Here we list the T-duality rules between the exotic Type IIB potentials with two isometry directions (on the left-hand side) and the IIA ones with the k isometry direction. These exotic fields correspond to some of the fields mentioned in [18].

$$
\begin{aligned}
\mathrm{i}_k\mathrm{i}_l N_7 &= \mathrm{i}_k\mathrm{i}_z N_7 - 5\mathrm{i}_k\mathrm{i}_z C_5(\mathrm{i}_k C_3 - 2\mathrm{i}_k\mathrm{i}_z C_3\frac{g_z}{g_{zz}}) - 5(\mathrm{i}_k C_5 - 4\mathrm{i}_k\mathrm{i}_z C_5\frac{g_z}{g_{zz}})\mathrm{i}_k\mathrm{i}_z C_3 \qquad (A.4)\\
\mathrm{i}_l\mathcal{N}_7 &= B_6 + 6\mathrm{i}_z B_6\frac{g_z}{g_{zz}}\\
\mathrm{i}_k\mathrm{i}_l N_8 &= \mathrm{i}_k\mathrm{i}_z N_8 - 6\mathrm{i}_k\mathrm{i}_z C_3(\mathrm{i}_k B_6 + 5\mathrm{i}_k\mathrm{i}_z B_6\frac{g_z}{g_{zz}}) - 15(\mathrm{i}_k C_3 + 2\mathrm{i}_k\mathrm{i}_z C_3\frac{g_z}{g_{zz}})\mathrm{i}_k\mathrm{i}_z B_6\\
&\quad +20\mathrm{i}_k\mathrm{i}_z C_3(C_3 - 3\mathrm{i}_z C_3\frac{g_z}{g_{zz}})(\mathrm{i}_k C_3 - 2\mathrm{i}_k\mathrm{i}_z C_3\frac{g_z}{g_{zz}}) + 30\mathrm{i}_z C_3(\mathrm{i}_k C_3 - 2\mathrm{i}_k\mathrm{i}_z C_3\frac{g_z}{g_{zz}})^2\\
\mathrm{i}_k\mathrm{i}_l\mathcal{N}_8 &= \mathrm{i}_k N_7 - 6\mathrm{i}_k\mathrm{i}_z N_7\frac{g_z}{g_{zz}}\\
\mathrm{i}_k\mathrm{i}_l\mathcal{N}_9 &= \mathrm{i}_k N_8 + 7\mathrm{i}_k\mathrm{i}_z N_8\frac{g_z}{g_{zz}}\\
\mathrm{i}_k\mathrm{i}_l N_9 &= \mathrm{i}_k\mathrm{i}_z N_9 - 7\mathrm{i}_k\mathrm{i}_z C_3(\mathrm{i}_k N_7 - 6\mathrm{i}_k\mathrm{i}_z N_7\frac{g_z}{g_{zz}}) - 35\mathrm{i}_k\mathrm{i}_z C_5(\mathrm{i}_k C_3 - 2\mathrm{i}_k\mathrm{i}_z C_3\frac{g_z}{g_{zz}})^2\\
\mathrm{i}_k\mathrm{i}_l N_{10} &= \mathrm{i}_k N_9 - 8\mathrm{i}_k\mathrm{i}_z N_9\frac{g_z}{g_{zz}}\\
\mathrm{i}_k\mathrm{i}_l\mathcal{N}_{10} &= \mathrm{i}_k\mathrm{i}_z N_{10} - 8\mathrm{i}_k\mathrm{i}_z C_3(\mathrm{i}_k N_8 + 7\mathrm{i}_k\mathrm{i}_z N_8\frac{g_z}{g_{zz}}) - 28(\mathrm{i}_k C_3 - 2\mathrm{i}_k\mathrm{i}_z C_3\frac{g_z}{g_{zz}})\mathrm{i}_k\mathrm{i}_z N_8\\
&\quad +168\mathrm{i}_k\mathrm{i}_z C_3(\mathrm{i}_k C_3 - 2\mathrm{i}_k\mathrm{i}_z C_3\frac{g_z}{g_{zz}})(\mathrm{i}_k B_6 - 5\mathrm{i}_k\mathrm{i}_z B_6\frac{g_z}{g_{zz}})\\
&\quad +210(\mathrm{i}_k C_3 - 2\mathrm{i}_k\mathrm{i}_z C_3\frac{g_z}{g_{zz}})^2\mathrm{i}_k\mathrm{i}_z B_6 - 210\mathrm{i}_k\mathrm{i}_z C_3(\mathrm{i}_k C_3 - 2\mathrm{i}_k\mathrm{i}_z C_3\frac{g_z}{g_{zz}})^2(C_3 - 3\mathrm{i}_z C_3\frac{g_z}{g_{zz}})\\
&\quad -315(\mathrm{i}_k C_3 - 2\mathrm{i}_k\mathrm{i}_z C_3\frac{g_z}{g_{zz}})^3\mathrm{i}_z C_3.
\end{aligned}
$$

As the last T-duality list this are the rules for the IIB exotic potentials dualized along the k direction. These are quite similar to the previous ones, as expected, but some of the fields are interchanged or have another sign.

$$
\begin{aligned}
\mathrm{i}_l\mathrm{i}_k\mathcal{N}_7 &= \mathrm{i}_l\mathrm{i}_z N_7 - 5\mathrm{i}_l\mathrm{i}_z C_5(\mathrm{i}_l C_3 - 2\mathrm{i}_l\mathrm{i}_z C_3\frac{g_z}{g_{zz}}) - 5(\mathrm{i}_l C_5 - 4\mathrm{i}_l\mathrm{i}_z C_5\frac{g_z}{g_{zz}})\mathrm{i}_l\mathrm{i}_z C_3 \qquad (A.5)\\
\mathrm{i}_k N_7 &= B_6 + 6\mathrm{i}_z B_6\frac{g_z}{g_{zz}}\\
-\mathrm{i}_l\mathrm{i}_k N_8 &= \mathrm{i}_l\mathrm{i}_z N_8 - 6\mathrm{i}_l\mathrm{i}_z C_3(\mathrm{i}_l B_6 + 5\mathrm{i}_l\mathrm{i}_z B_6\frac{g_z}{g_{zz}}) - 15(\mathrm{i}_l C_3 - 2\mathrm{i}_l\mathrm{i}_z C_3\frac{g_z}{g_{zz}})\mathrm{i}_l\mathrm{i}_z B_6\\
&\quad +20\mathrm{i}_l\mathrm{i}_z C_3(C_3 - 3\mathrm{i}_z C_3\frac{g_z}{g_{zz}})(\mathrm{i}_l C_3 - 2\mathrm{i}_l\mathrm{i}_z C_3\frac{g_z}{g_{zz}}) + 30\mathrm{i}_z C_3(\mathrm{i}_l C_3 - 2\mathrm{i}_l\mathrm{i}_z C_3\frac{g_z}{g_{zz}})^2\\
-\mathrm{i}_l\mathrm{i}_k\mathcal{N}_8 &= \mathrm{i}_l N_7 - 6\mathrm{i}_l\mathrm{i}_z N_7\frac{g_z}{g_{zz}}\\
-\mathrm{i}_l\mathrm{i}_k\mathcal{N}_9 &= \mathrm{i}_l N_8 + 7\mathrm{i}_l\mathrm{i}_z N_8\frac{g_z}{g_{zz}}\\
-\mathrm{i}_l\mathrm{i}_k N_9 &= \mathrm{i}_l\mathrm{i}_z N_9 - 7\mathrm{i}_l\mathrm{i}_z C_3(\mathrm{i}_l N_7 - 6\mathrm{i}_l\mathrm{i}_z N_7\frac{g_z}{g_{zz}}) - 35\mathrm{i}_l\mathrm{i}_z C_5(\mathrm{i}_l C_3 - 2\mathrm{i}_l\mathrm{i}_z C_3\frac{g_z}{g_{zz}})^2\\
\mathrm{i}_l\mathrm{i}_k N_{10} &= \mathrm{i}_l N_9 - 8\mathrm{i}_l\mathrm{i}_z N_9\frac{g_z}{g_{zz}}\\
\mathrm{i}_l\mathrm{i}_k\mathcal{N}_{10} &= \mathrm{i}_l\mathrm{i}_z N_{10} - 8\mathrm{i}_l\mathrm{i}_z C_3(\mathrm{i}_l N_8 + 7\mathrm{i}_l\mathrm{i}_z N_8\frac{g_z}{g_{zz}}) - 28(\mathrm{i}_l C_3 - 2\mathrm{i}_l\mathrm{i}_z C_3\frac{g_z}{g_{zz}})\mathrm{i}_l\mathrm{i}_z N_8\\
&\quad +168\mathrm{i}_l\mathrm{i}_z C_3(\mathrm{i}_l C_3 - 2\mathrm{i}_l\mathrm{i}_z C_3\frac{g_z}{g_{zz}})(\mathrm{i}_l B_6 - 5\mathrm{i}_l\mathrm{i}_z B_6\frac{g_z}{g_{zz}})\\
&\quad +210(\mathrm{i}_l C_3 - 2\mathrm{i}_l\mathrm{i}_z C_3\frac{g_z}{g_{zz}})^2\mathrm{i}_l\mathrm{i}_z B_6 - 210\mathrm{i}_l\mathrm{i}_z C_3(C_3 - 3\mathrm{i}_z C_3\frac{g_z}{g_{zz}})(\mathrm{i}_l C_3 - 2\mathrm{i}_l\mathrm{i}_z C_3\frac{g_z}{g_{zz}})^2\\
&\quad -315(\mathrm{i}_l C_3 - 2\mathrm{i}_l\mathrm{i}_z C_3\frac{g_z}{g_{zz}})^3\mathrm{i}_z C_3.
\end{aligned}
$$

B Gauge variations

Gauge variations and coordinate transformation in the eleven-dimensional theory:

$$\begin{aligned}
\delta\hat{k}^{-2}\hat{k}_1 &= \partial\hat{\Lambda}_0 \\
\delta\hat{C}_3 &= 3\partial\hat{\Lambda}_2 \\
\delta\hat{C}_6 &= 6\partial\hat{\Lambda}_5 + 30\hat{C}_3\partial\hat{\Lambda}_2 \\
\delta\mathrm{i}_k\hat{C}_8 &= -7\partial(\mathrm{i}_k\hat{\Lambda}_7) - 105\mathrm{i}_k\hat{C}_3\partial(\mathrm{i}_k\hat{\Lambda}_5) + 210(\mathrm{i}_k\hat{C}_3)^2\partial\hat{\Lambda}_2 + 140\mathrm{i}_k\hat{C}_3\hat{C}_3\partial(\mathrm{i}_k\hat{\Lambda}_2) \\
\delta(\mathrm{i}_k\hat{C}_{10}) &= -9\partial(\mathrm{i}_k\hat{\Lambda}_9) - 252\mathrm{i}_k\hat{C}_3\partial(\mathrm{i}_k\hat{\Lambda}_7) - 1890(\mathrm{i}_k\hat{C}_3)^2\partial(\mathrm{i}_k\hat{\Lambda}_5) \\
&\quad +2835(\mathrm{i}_k\hat{C}_3)^3\partial\hat{\Lambda}_2 + 1890\hat{C}_3(\mathrm{i}_k\hat{C}_3)^2\partial(i_k\hat{\Lambda}_2).
\end{aligned} \tag{B.1}$$

Gauge variations in type IIA:

$$\begin{aligned}
\delta k^{-2}k_1 &= \partial\xi_0 \\
\delta B_2 &= 2\partial\Sigma_1 \\
\delta C_{2n+1} &= \sum_{p=0}^{n}\frac{(2n+1)!}{2^p p!(2n-2p)!}(B_2)^p\partial\Lambda_{2n-2p} \\
\delta B_6 &= 6\partial\Sigma_5 + 30C_3\partial\Lambda_2 - 6C_5\partial\Lambda_0 + 30C_3B_2\partial\Lambda_0 \\
\delta\mathrm{i}_kN_7 &= -6\partial\mathrm{i}_k\xi_6 - 30\mathrm{i}_kB_2\partial(\mathrm{i}_k\Sigma_5) - 60\mathrm{i}_kC_3\partial(\mathrm{i}_k\Lambda_4) - 180\mathrm{i}_kC_3B_2\partial(\mathrm{i}_k\Lambda_2) + 180\mathrm{i}_kC_3\mathrm{i}_kB_2\partial\Lambda_2 \\
&\quad +180\mathrm{i}_kC_3B_2\mathrm{i}_kB_2\partial\Lambda_0 \\
\delta\mathrm{i}_kN_8 &= -7\partial(\mathrm{i}_k\xi_7) - 105\mathrm{i}_kC_3\partial(\mathrm{i}_k\Sigma_5) + 140\mathrm{i}_kC_3C_3\partial(\mathrm{i}_k\Lambda_2) + 210\mathrm{i}_kC_3\mathrm{i}_kC_3\partial\Lambda_2 \\
&\quad -7\mathrm{i}_kN_7\partial\Lambda_0 - 140\mathrm{i}_kC_3C_3\mathrm{i}_kB_2\partial\Lambda_0 + 210\mathrm{i}_kC_3\mathrm{i}_kC_3B_2\partial\Lambda_0 \\
\delta\mathrm{i}_kN_9 &= -8\partial(\mathrm{i}_k\xi_8) - 56\mathrm{i}_kB_2\partial(\mathrm{i}_k\xi_7) - 168\mathrm{i}_kC_3\partial(\mathrm{i}_k\xi_6) - 840\mathrm{i}_kC_3\mathrm{i}_kC_3\partial(\mathrm{i}_k\Lambda_4) \\
&\quad -840\mathrm{i}_kC_3\mathrm{i}_kB_2\partial(\mathrm{i}_k\Sigma_5) + 2520(\mathrm{i}_kC_3)^2\mathrm{i}_kB_2\partial\Lambda_2 \\
&\quad -2520B_2(\mathrm{i}_kC_3)^2\partial(\mathrm{i}_k\Lambda_2) + 2520(\mathrm{i}_kC_3)^2B_2\mathrm{i}_kB_2\partial\Lambda_0 \\
\delta\mathrm{i}_kN_{10} &= -9\partial(\mathrm{i}_k\zeta_9) - 252\mathrm{i}_kC_3\partial(i_k\xi_7) - 1890(\mathrm{i}_kC_3)^2\partial(\mathrm{i}_k\Sigma_5) + 2835(\mathrm{i}_kC_3)^3\partial\Lambda_2 \\
&\quad +1890C_3(\mathrm{i}_kC_3)^2\partial(\mathrm{i}_k\Lambda_2) - 9\mathrm{i}_kN_9\partial\Lambda_0 \\
&\quad +2835(\mathrm{i}_kC_3)^3B_2\partial\Lambda_0 - 1890C_3(\mathrm{i}_kC_3)^2i_kB_2\partial\Lambda_0.
\end{aligned} \tag{B.2}$$

Gauge variations in type IIB:

$$\begin{aligned}
\delta B_2 &= 2\partial\Sigma_1 \\
\delta C_{2n} &= \sum_{p=0}^{n-1}\frac{(2n)!}{2^p p!(2n-2p-1)!}B_2^p\partial\Lambda_{2n-2p-1} \\
\delta B_6 &= 6\partial\Sigma_5 - 30C_4\partial\Lambda_1 \\
\delta Q_8 &= 8\partial\lambda_7 - 56B_6\partial\Lambda_1 \\
\delta Q_{10} &= 10\partial\lambda_9 - 90Q_8\partial\Lambda_1 \\
\delta\mathrm{i}_kN_7 &= -6\partial\mathrm{i}_k\xi_6 - 30\mathrm{i}_kC_4\partial(\mathrm{i}_k\Lambda_3) + 6\mathrm{i}_kC_6\partial(\mathrm{i}_k\Lambda_1) - 30\mathrm{i}_kC_4B_2\partial(\mathrm{i}_k\Lambda_1) - 30\mathrm{i}_kB_2\partial(\mathrm{i}_k\Sigma_5) \\
&\quad +60\mathrm{i}_kC_4\mathrm{i}_kB_2\partial\Lambda_1 \\
\delta\mathrm{i}_l\mathcal{N}_7 &= -6\partial\mathrm{i}_l\zeta_6 - 30\mathrm{i}_lC_4\partial(\mathrm{i}_l\Lambda_3) + 6\mathrm{i}_lC_6\partial(\mathrm{i}_l\Lambda_1) - 30\mathrm{i}_lC_4B_2\partial(\mathrm{i}_l\Lambda_1) - 30\mathrm{i}_lB_2\partial(\mathrm{i}_l\Sigma_5) \\
&\quad +60\mathrm{i}_lC_4\mathrm{i}_lB_2\partial\Lambda_1 \\
\delta(\mathrm{i}_k\mathrm{i}_lN_8) &= 6\partial(\mathrm{i}_k\mathrm{i}_l\xi_7) - 30\mathrm{i}_k\mathrm{i}_lB_6\partial(\mathrm{i}_k\mathrm{i}_l\Lambda_3) - 6\mathrm{i}_k\mathrm{i}_l\mathcal{N}_7\partial(\mathrm{i}_k\Lambda_1) + 6\mathrm{i}_k\mathrm{i}_lN_7\partial(\mathrm{i}_l\Lambda_1) \\
&\quad -30\mathrm{i}_k\mathrm{i}_lB_6\mathrm{i}_lB_2\partial(\mathrm{i}_k\Lambda_1) + 30\mathrm{i}_k\mathrm{i}_lB_6\mathrm{i}_kB_2\partial(\mathrm{i}_l\Lambda_1) \\
&\quad +30\mathrm{i}_k\mathrm{i}_lC_4\mathrm{i}_lC_4\partial(\mathrm{i}_k\Lambda_1) - 30\mathrm{i}_k\mathrm{i}_lC_4\mathrm{i}_kC_4\partial(\mathrm{i}_l\Lambda_1) + 90(\mathrm{i}_k\mathrm{i}_lC_4)^2\partial\Lambda_1 \\
\delta(\mathrm{i}_k\mathrm{i}_l\mathcal{N}_8) &= 6\partial(\mathrm{i}_k\mathrm{i}_l\zeta_7) + 30\mathrm{i}_kB_2\partial(\mathrm{i}_k\mathrm{i}_l\zeta_6) - 30\mathrm{i}_lB_2\partial(\mathrm{i}_k\mathrm{i}_l\xi_6) - 120\mathrm{i}_kB_2\mathrm{i}_lB_2\partial(\mathrm{i}_k\mathrm{i}_l\Sigma_5)
\end{aligned} \tag{B.3}$$

$$
\begin{aligned}
& +60\mathrm{i}_k\mathrm{i}_l C_4\partial(\mathrm{i}_k\mathrm{i}_l\Lambda_5)+180\mathrm{i}_k\mathrm{i}_l C_4\mathrm{i}_l B_2\partial\mathrm{i}_k\Lambda_3-180\mathrm{i}_k\mathrm{i}_l C_4\mathrm{i}_k B_2\partial(\mathrm{i}_l\Lambda_3)\\
& +180\mathrm{i}_k\mathrm{i}_l C_4 B_2\partial(\mathrm{i}_k\mathrm{i}_l\Lambda_3)-180\mathrm{i}_k\mathrm{i}_l C_4 B_2\mathrm{i}_k B_2\partial(i_l\Lambda_1)+180\mathrm{i}_k\mathrm{i}_l C_4 B_2\mathrm{i}_l B_2\partial(\mathrm{i}_k\Lambda_1)\\
& +360\mathrm{i}_k\mathrm{i}_l C_4\mathrm{i}_k B_2\mathrm{i}_l B_2\partial\Lambda_1\\
\delta(\mathrm{i}_k\mathrm{i}_l N_9) \;=\;& 7\partial(\mathrm{i}_k\mathrm{i}_l\xi_8)-42\mathrm{i}_k B_2\partial(\mathrm{i}_k\mathrm{i}_l\xi_7)+105\mathrm{i}_k\mathrm{i}_l C_4\partial(\mathrm{i}_k\mathrm{i}_l\xi_6)-420\mathrm{i}_k\mathrm{i}_l C_4\mathrm{i}_k B_2\partial(\mathrm{i}_k\mathrm{i}_l\Sigma_5)\\
& -140\mathrm{i}_k\mathrm{i}_l C_4\mathrm{i}_k C_4\partial(\mathrm{i}_k\mathrm{i}_l\Lambda_3)-210(\mathrm{i}_k\mathrm{i}_l C_4)^2\partial(\mathrm{i}_k\Lambda_3)\\
& +7\mathrm{i}_k\mathrm{i}_l\mathcal{N}_8\partial(\mathrm{i}_k\Lambda_1)+140\mathrm{i}_k\mathrm{i}_l C_4\mathrm{i}_k C_4\mathrm{i}_k B_2\partial(\mathrm{i}_l\Lambda_1)\\
& -140\mathrm{i}_k\mathrm{i}_l C_4\mathrm{i}_k C_4\mathrm{i}_l B_2\partial(\mathrm{i}_k\Lambda_1)-210(\mathrm{i}_k\mathrm{i}_l C_4)^2 B_2\partial(\mathrm{i}_k\Lambda_1)+420(\mathrm{i}_k\mathrm{i}_l C_4)^2\mathrm{i}_k B_2\partial\Lambda_1\\
\delta(\mathrm{i}_k\mathrm{i}_l\mathcal{N}_9) \;=\;& 7\partial(\mathrm{i}_k\mathrm{i}_l\zeta_8)-42\mathrm{i}_l B_2\partial(\mathrm{i}_k\mathrm{i}_l\xi_7)+105\mathrm{i}_k\mathrm{i}_l C_4\partial(\mathrm{i}_k\mathrm{i}_l\zeta_6)-420\mathrm{i}_k\mathrm{i}_l C_4\mathrm{i}_l B_2\partial(\mathrm{i}_k\mathrm{i}_l\Sigma_5)\\
& -140\mathrm{i}_k\mathrm{i}_l C_4\mathrm{i}_l C_4\partial(\mathrm{i}_k\mathrm{i}_l\Lambda_3)-210(\mathrm{i}_k\mathrm{i}_l C_4)^2\partial(\mathrm{i}_l\Lambda_3)\\
& +7\mathrm{i}_k\mathrm{i}_l\mathcal{N}_8\partial(\mathrm{i}_l\Lambda_1)+140\mathrm{i}_k\mathrm{i}_l C_4\mathrm{i}_l C_4\mathrm{i}_k B_2\partial(\mathrm{i}_l\Lambda_1)\\
& -140\mathrm{i}_k\mathrm{i}_l C_4\mathrm{i}_l C_4\mathrm{i}_l B_2\partial(\mathrm{i}_k\Lambda_1)-210(\mathrm{i}_k\mathrm{i}_l C_4)^2 B_2\partial(\mathrm{i}_l\Lambda_1)+420(\mathrm{i}_k\mathrm{i}_l C_4)^2\mathrm{i}_l B_2\partial\Lambda_1\\
\delta(\mathrm{i}_k\mathrm{i}_l N_{10}) \;=\;& 8\partial(\mathrm{i}_k\mathrm{i}_l\xi_9)-56\mathrm{i}_l B_2\partial(\mathrm{i}_k\mathrm{i}_l\zeta_8)+56\mathrm{i}_k B_2\partial(\mathrm{i}_k\mathrm{i}_l\xi_8)-336\mathrm{i}_k B_2\mathrm{i}_l B_2\partial(\mathrm{i}_k\mathrm{i}_l\xi_7)\\
& +168\mathrm{i}_k\mathrm{i}_l C_4\partial(\mathrm{i}_k\mathrm{i}_l\zeta_7)+840\mathrm{i}_k\mathrm{i}_l C_4\mathrm{i}_k B_2\partial(\mathrm{i}_k\mathrm{i}_l\zeta_6)-840\mathrm{i}_k\mathrm{i}_l C_4\mathrm{i}_l B_2\partial(\mathrm{i}_k\mathrm{i}_l\xi_6)\\
& +840(\mathrm{i}_k\mathrm{i}_l C_4)^2\partial(\mathrm{i}_k\mathrm{i}_l\Lambda_5)-3360\mathrm{i}_k\mathrm{i}_l C_4\mathrm{i}_k B_2\mathrm{i}_l B_2\partial(\mathrm{i}_k\mathrm{i}_l\Sigma_5)+2520(\mathrm{i}_k\mathrm{i}_l C_4)^2\mathrm{i}_l B_2\partial(\mathrm{i}_k\Lambda_3)\\
& -2520(\mathrm{i}_k\mathrm{i}_l C_4)^2\mathrm{i}_k B_2\partial(\mathrm{i}_l\Lambda_3)+2520(\mathrm{i}_k\mathrm{i}_l C_4)^2 B_2\partial(\mathrm{i}_k\mathrm{i}_l\Lambda_3)\\
& +5040(\mathrm{i}_k\mathrm{i}_l C_4)^2\mathrm{i}_k B_2\mathrm{i}_l B_2\partial\Lambda_1+2520(\mathrm{i}_k\mathrm{i}_l C_4)^2 B_2\mathrm{i}_l B_2\partial(\mathrm{i}_k\Lambda_1)\\
& -2520(\mathrm{i}_k\mathrm{i}_l C_4)^2 B_2\mathrm{i}_k B_2\partial(\mathrm{i}_l\Lambda_1)\\
\delta(\mathrm{i}_k\mathrm{i}_l\mathcal{N}_{10}) \;=\;& 8\partial(\mathrm{i}_k\mathrm{i}_l\zeta_9)-56\mathrm{i}_k\mathrm{i}_l N_8\partial(\mathrm{i}_k\mathrm{i}_l\Lambda_3)-56\mathrm{i}_k\mathrm{i}_l N_8\mathrm{i}_l B_2\partial(\mathrm{i}_k\Lambda_1)+56\mathrm{i}_k\mathrm{i}_l N_8\mathrm{i}_k B_2\partial(\mathrm{i}_l\Lambda_1)\\
& -8\mathrm{i}_k\mathrm{i}_l\mathcal{N}_9\partial(\mathrm{i}_k\Lambda_1)+8\mathrm{i}_k\mathrm{i}_l N_9\partial(\mathrm{i}_l\Lambda_1)-168\mathrm{i}_k\mathrm{i}_l N_7\mathrm{i}_k\mathrm{i}_l C_4\partial(\mathrm{i}_l\Lambda_1)\\
& +168\mathrm{i}_k\mathrm{i}_l\mathcal{N}_7\mathrm{i}_k\mathrm{i}_l C_4\partial(\mathrm{i}_k\Lambda_1)-280\mathrm{i}_l C_4(\mathrm{i}_k\mathrm{i}_l C_4)^2\partial(\mathrm{i}_k\Lambda_1)\\
& -280\mathrm{i}_k C_4(\mathrm{i}_k\mathrm{i}_l C_4)^2\partial(\mathrm{i}_l\Lambda_1)-840(\mathrm{i}_k\mathrm{i}_l C_4)^3\partial\Lambda_1.
\end{aligned}
$$

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